

# The Investigation and Application of Tsallis Random Generator

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## ABSTRACT

Tsallis distribution was proposed by C. Tsallis in 1996 to solve the slow convergence problem of simulated annealing. It is shown that Tsallis's generalized simulated annealing is much faster than the classical simulated annealing ("Boltzmann machine") and fast simulated annealing ("Cauchy machine"). However, Tsallis distribution is very complicated and its random variable could not be generated by ordinary simulation techniques such as inversion and rejection methods. Tsallis adopts algorithm of R. N. Mantegna (1994) to produce a Tsallis random number generator. This method has many problems, however. First it could generate complex number when the parameter is near by 1.4. Second, when it is generated using Monte Carlo simulation, its histogram is not identical with the corresponding theoretical probability density (PDF). We plan to come out with a better Tsallis random number generator which can match the Tsallis's PDF in most cases of its parameter's ranges.

Keywords : Tsallis distribution ; Simulated Annealing ; Monte Carlo simulation ; random number generator

## Table of Contents

目錄 封面內頁 簽名頁 授權書 .....	iii 中文摘要 .....
..... iv 英文摘要 .....	v 誌謝 .....
..... vi 目錄 .....	vii 圖目錄 .....
..... ix 表目錄 .....	xi 第一章 緒論 .....
..... 1 1.1 研究背景與動機 .....	1 1.2 研究目的 .....
..... 2 1.3 研究範圍與限制 .....	2 1.4 研究流程 .....
..... 3 1.5 論文章節架構 .....	6 第二章 文獻探討 .....
..... 8 2.1 Inversion .....	9 2.2 Rejection .....
..... 11 2.3 Composition .....	13 2.4 Box Muller's method .....
..... Ingenious Method for Symmetric Stable Variate..	18 2.6 小結 .....
..... 方法與流程 .....	21 3.1 Tsallis隨機變數產生器 .....
..... statistic檢定.....	21 3.2 Kolmogorov's .....
..... 38 第四章 結論與建議 .....	44 4.1 結論 .....
..... 44 4.2 建議及後續研究之方向.....	44 參考文獻 .....
..... 46 附錄一 Algorithm TR.....	49 附錄二 Chamber stable .....
..... variate.....	51 附錄三 Chambers和Levy之關係式.....
..... 53 附錄四 C.Tsallis' Algorithm .....	53 附錄四 C.Tsallis' Algorithm .....
..... 55 附錄五 Probability distribution of .....	57

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