

# 2-Disjoint Geodesic Bipancyclicity of Hypercubes

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## ABSTRACT

Let  $G = (V, E)$  be a graph. For any two vertices  $u, v \in V(G)$ , a cycle  $C$  is called  $(u, v)$ -geodesic if there exists a  $u$ - $v$  shortest path of  $G$  lying on  $C$ . A bipartite graph  $G$  is called geodesic bipancyclic if for any two vertices  $u, v \in V$ , there exists a  $(u, v)$ -geodesic cycle of every even length ranging from  $\max\{2d(u, v), 4\}$  to  $|V|$ . In this thesis, we first show that the hypercube  $Q_n$  for  $n \geq 4$  is geodesic bipancyclic when it has two adjacent fault vertices. Then we prove that  $Q_n$  is 2-disjoint geodesic bipancyclicity for  $n \geq 4$ . That is, given any four vertices  $u, v, x, y$  without forming  $u, v, x, y$  paths, and given any even integers  $l_1, l_2$  such that  $l_1 + l_2 \leq 2n$ ,  $l_1 \geq \min\{2d(u, v) + 2, 2n\}$ , and  $l_2 \geq \min\{2d(x, y) + 2, 2n\}$ , there exist two disjoint cycles  $C_1$  and  $C_2$  in  $Q_n$  such that  $C_1$  is a  $(u, v)$ -geodesic cycle of length  $l_1$ , and  $C_2$  is a  $(x, y)$ -geodesic cycle of length  $l_2$ .

Keywords : Geodesic Bipancyclic, 2 – Disjoint Geodesic Bipancyclicity, Hypercube

## Table of Contents

封面內頁	
簽名頁	
授權書	iii
ABSTRACT	iv
中文摘要	v
誌謝	vi
Contents	vii
List of Figures	viii

Chapter 1. Introduction	1
Chapter 2. Preliminaries	4
Chapter 3. Adjacent fault geodesic bipancyclicity	6
Chapter 4. 2-disjoint Geodesic embedding	19
Chapter 5. Conclusion	34
Reference	35

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