

# 超立方體之互斥最短路徑泛圈性研究

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## 摘要

在一個圖形  $G = (V, E)$  中，給定任兩點  $u$  及  $v$ ，若一迴圈  $C$  上存在一條  $u$  到  $v$  的最短路徑，就稱  $C$  為  $(u, v)$ -geodesic。令圖形  $G$  為一個雙分圖，給任兩點  $u, v$ ，若存在任意偶數長度（範圍從  $\max\{2d(u, v), 4\}$  到  $|V|$ ）的  $(u, v)$ -geodesic 迴圈，此圖形  $G$  被稱之為最短路徑偶泛圈性 (geodesic bipancyclic)。在這篇論文裡，我們首先證明了超立方體  $Q_n$  ( $n \geq 4$ ) 在壞一對相鄰點後仍有最短路徑偶泛圈性性質；接下來證明  $Q_n$  ( $n \geq 4$ ) 有二互斥最短路徑偶泛圈性 (2-disjoint geodesic bipancyclic) 性質，也就是說，當任給四點  $u, v, x, y$  且它們不形成  $\{x, v\}$  或  $\{y, v\}$  或  $\{u, y\}$  或  $\{v, y\}$  路徑時，會存在兩個不相交的  $(u, v)$ -geodesic 迴圈  $C_1$  和  $(x, y)$ -geodesic 迴圈  $C_2$ ，其中  $C_1$  長度為  $l_1$ ， $C_2$  長度為  $l_2$ ，且  $l_1$  及  $l_2$  是任意符合  $l_1 + l_2 \geq 2n$  且  $l_1 \geq \min\{2d(u, v) + 2, 2n\}$  及  $l_2 \geq \min\{2d(x, y) + 2, 2n\}$  等三個條件的的偶數。

關鍵詞：最短路徑偶泛圈性、二互斥最短路徑偶泛圈性、超立方體

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