

# Vibration analysis of beams with an attached Spring-Mass system

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## ABSTRACT

In this study, an analytical method that permits the efficient calculation of the dynamics of a beam with an attached spring-mass system. First, assuming the beam obeying the Euler-Bernoulli beam theory, the equations of motion of the beam and the attached mass are derived. By using the compatibility requirements of the spring attached point and the transfer matrix method, the characteristic equation of the system can be obtained. From the system characteristic equation, the eigensolutions (natural frequencies and mode shapes) of this system can be determined. Some numerical results are calculated and compared with the previous researches and an experimental method was used to validate the theoretical model.

Keywords : Euler-Bernoulli, transfer matrix, characteristic equation, eigensolution, natural frequency, mode shape.

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