

# 具彈簧：質量系統之樑結構之振動分析 = Vibration analysis of beams with an attached Spring-Mass system

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## 摘要

本研究是利用理論數值分析之方法，探討一具彈簧 - 質量系統之樑結構之動態特性。首先，應用Euler-Bernoulli 樑理論推導出樑系統之運動方程式及所懸掛質量之運動方程式，再利用懸吊點之匹配要求(compatibility requirement)及轉移矩陣(transfer matrix)之方法，並導出此具單一彈簧 - 質量系統之樑系統的特徵方程式，由此可以計算出此樑結構系統的特徵解(自然頻率及模態)。本研究理論分析結果並與國內外文獻互相驗證。最後，並以實驗方法對此理論模式加以驗證。

關鍵詞：Euler-Bernoulli、轉移矩陣、特徵方程式、特徵值、自然頻率、模態

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