

# Pancyclicity of Hypercube Variants

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## ABSTRACT

Let  $G = (V, E)$  be a graph. For any two vertices  $x, y \in V$ , a cycle  $C$  is called  $(x, y)$ -geodesic if there exists a shortest  $x$ - $y$  path of  $G$  lies on  $C$ . A graph  $G$  is weakly-geodesic  $r$ -pancyclic if for any two vertices  $x, y \in V$ , there exists a  $(x, y)$ -geodesic of every length ranging from  $\max\{2d(x, y), r\}$  to  $2d(x, y) + r$ . A graph  $G$  is geodesic  $r$ -pancyclic if for any two vertices  $x, y \in V$  and any shortest  $x$ - $y$  path  $P$ , there exists a  $(x, y)$ -geodesic  $l$ -cycle containing  $P$ , where  $l$  is any integer between  $\max\{2d(x, y), r\}$  to  $2d(x, y) + r$  inclusive. A bipartite graph  $G$  is weakly-geodesic  $(+r)$ -bipancyclic if for any two vertices  $x, y \in V$ , there exists a  $(x, y)$ -geodesic cycle of every even length ranging from  $2d(x, y) + r$  to  $2d(x, y) + 2r$ . In this thesis, we shall show that the  $k$ -ary  $n$ -cube is geodesic 3-pancyclic when  $k = 3$ , and weakly-geodesic  $(+2)$ -bipancyclic when  $k$  is even. For any two vertices  $x, y \in V$ , a cycle  $C$  is called  $(x, y)$ -balanced if the distance  $d_C(x, y) = \max\{d_C(u, v) \mid u, v \in V\}$  when  $G$  is not bipartite, and  $d_C(x, y) = \max\{d_C(u, v) \mid x, u \in A, y, v \in B\}$  when  $G$  is bipartite with bipartitions  $A, B$ , and  $x \in A, y \in B$ . A graph  $G$  is balanced  $r$ -pancyclic if for any two vertices  $x, y \in V$ , there exists a  $(x, y)$ -balanced cycle of every length ranging from  $\max\{2d(x, y), r\}$  to  $2d(x, y) + r$ . A graph  $G$  is balanced  $(+r)$ -bipancyclic if for any two vertices  $x, y \in V$ , there exists a  $(x, y)$ -balanced cycle of every even length ranging from  $2d(x, y) + r$  to  $2d(x, y) + 2r$ . In this thesis, we shall show that the  $k$ -ary  $n$ -cube is balanced 5-pancyclic when  $k = 3$ , and balanced  $(+2)$ -bipancyclic when  $k > 2$  is even.

Keywords : geodesic pancyclic ; balanced pancyclic ;  $k$ -ary  $n$ -cube

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