

The Study of Fault Tolerance for Hamiltonicity of Node Expansion on Hypercube

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ABSTRACT

In this thesis, we construct the variant of hypercube $X(Q_n, \{x_b, x_w\})$ with node expansion on one black node x_b and one white node x_w of hypercube $Q_n = (V_b \cup V_w, E)$. We investigate the fault tolerance for multi-spanning disjoint paths of complete graph K_n . Let $F \subseteq (V \cup E)$ be a faulty set on complete graph K_n . We prove $K_n - F$ is Hamiltonian connected for $|F| \leq n - 4$. Secondly, we show that there exist m spanning disjoint paths in $K_n - F$ for $|F| \leq n - 2$ and $1 \leq m \leq \lfloor (n - |F|) / 2 \rfloor$. We thus prove that for any m pairs of fault-free vertices in $K_n - F$, there exist m spanning disjoint paths of $K_n - F$ for $|F| \leq n - 5$ and $2 \leq m \leq \lfloor (n - |F|) / 2 \rfloor$. Let $F = F_b \cup F_w \cup F'$ be the faulty set of $X(Q_n, \{x_b, x_w\})$ where $F_b \subseteq V_b$, $F_w \subseteq V_w$ and F' are disjoint sets. We show that $X(Q_n, \{x_b, x_w\}) - F$ is Hamiltonian if one of the following condition holds. (1). $|F_b| = |F_w| = 0$, $|F'| \leq n - 2$, (2). $0 < |F_b| = |F_w| \leq \lfloor n/4 \rfloor - 1$, $|F'| \leq n - 1 - 4|F_b|$, (3). $0 \leq |F_w| \leq |F_b| \leq \lfloor n/4 \rfloor - 2$, $|F'| \leq n - 3 - 4f_{\max}$, for $f_{\max} = \max\{|F_b|, |F_w|\}$. We thus derive that $X(Q_n, \{x_b, x_w\})$ is k -Hamiltonian for $k = \lfloor n/4 \rfloor - 2$. We furthermore show that $X(Q_n, \{x_b, x_w\})$ is k -Hamiltonian connected for $k \leq \lfloor (n-2)/4 \rfloor - 2$.

Keywords : Hypercube ; Node expansion ; Spanning disjoint path

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REFERENCES

- [1] Toru Araki and Yosuke Kikuchi, Hamiltonian laceability of bubble-sort graphs with edge faults," Information Sciences, pp.2679-2691, (2007)
- [2] Rostislav Caha and Vclav Koubek, Spanning multi-paths in Hypercubes," Discrete Mathematics, pp.2053-2066, (2007).
- [3] Y-Chuang Chen, Chang-Hsiung Tsai, Lih-Hsing Hsu, Jimmy J.M. Tan On some super Fault-tolerant Hamiltonian graphs," Applied Mathematics and Computation, pp.729-741, (2004)
- [4] Tomas Dvorak and Petr Gregor Hamiltonian Fault-tolerance of Hypercubes," Electronic Notes in Discrete Mathematics, pp.471-477, (2007)
- [5] Tomas Dvorak and Petr Gregor Hamiltonian paths with prescribed edges in Hypercubes," Discrete Mathematics, pp.1982-1998, (2007)
- [6] Jianxi Fan, Xiaola Lin, Yi Pan, Xiaohua Jia Optimal Fault-tolerant embedding of paths in twisted cubes," J. Parallel Distrib. Comput., pp.205-214, (2007)
- [7] Jung-Sheng Fu, Conditional fault Hamiltonicity of the complete graph," To appear Information Processing Letters, (2008)
- [8] Hong-Chun Hsu, Liang-Chih Chiang, Jimmy J.M. Tan and Lih-Hsing Hsu, Fault Hamiltonicity of augmented cubes," Parallel Computing, pp.131-145, (2005).
- [9] Wen-Tzeng Huang, Y.C. Chuang, J.M. Tan and L.H. Hsu, On the Fault-tolerant Hamiltonicity of faulty crossed cubes," IEICE Transaction on Fundamentals of Electronics, Communications and Computer Sciences, pp.1359- 1370, (2002).
- [10] Wen-Tzeng Huang, J. M. Tan, C. N. Hung, and L. H. Hsu, Fault-tolerant Hamiltonicity of twisted cubes," Journal of Parallel and Distributed Computing, pp.519-604, (2002).
- [11] Chun-Nan Hung, Lih-Hsing Hsu, and Ting-Yi Sung, On the Construction of Combined k -Fault-Tolerant Hamiltonian Graphs," NETWORKS, pp.165-170, (2001).
- [12] Hao-Shun Hung, Jung-Sheng Fu, and Gen-Huey Chen, Fault-free Hamiltonian cycles in crossed cubes with conditional link faults,"

Information Sciences, pp.5664-5674, (2007).

- [13] Chun-Nan Hung, Hsuan-Han Chang, and Guan-Yu Shi, Fault tolerance for Hamiltonian cycle of node expansion on Hypercube," National Computer Symposium, pp.621-626, (2007).
- [14] Chun-Nan Hung, Chi-Lai Liu, and Hsuan-Han Chang, Edge for tolerance for two spanning disjoint paths of Star network," Processing of the 25rd Workshop on Combinatorial Mathematics and Computational Theory, pp.375-384, (2008).
- [15] Chun-Nan Hung and Guan-Yu Shi, Vertex Fault tolerance for multiple spanning paths in Hypercube," Processing of the 24rd Workshop on Combinatorial Mathematics and Computational Theory, pp.241-250, (2007).
- [16] Sun-Yuan Hsieh, Che-Nan Kuo, Hamiltonian-connectivity and strongly Hamiltonian-laceability of folded Hypercubes," Computers and Mathematics with Applications, pp.1040-1044, (2007).
- [17] Sun-Yuan Hsieh, Gen-Huey Chen, Chin-Wen Ho, Hamiltonian-laceability of Star graphs," Networks, pp.225-232, (2000).
- [18] Tseng-Kuei Li, Jimmy J. M. Tan, Lih-Hsing Hsu, Hyper Hamiltonian laceability on edge fault Star graph," Information Sciences, pp.59-77, (2007).
- [19] Krishnendu Mukhopadhyaya, and Bhabani P. Sinha, Hamiltonian graphs with minimum number of edges for Fault-tolerant topologies," Information Processing Letters, pp.95-99, (1990).
- [20] Chong-Dae Park, Kyung-Yong Chwa, Hamiltonian properties on the class of Hypercube-like networks," Information Processing Letters, pp.11-17, (2005). {34{ [21] J.-H. Park, H.-C. Kim, H.-S. Lim, Fault-Hamiltonicity of Hypercube-like interconnection networks," IEEE International Parallel and Distributed Processing Symposium, pp.60a, (2005).
- [22] Youcef Saad and Martin H. Schultz, Topological properties of Hypercubes," IEEE Transactions On Computers, pp.867-872, (1998).
- [23] Chang-Hsiung Tsai, Jimmy J.M. Tan, Tyne Liang, Lih-Hsing Hsu, Fault-tolerant Hamiltonian laceability of Hypercubes," Information Processing Letters, pp.301-306, (2002).
- [24] Aniruddha S. Vaidya, P. S. Nagendra Rao, S. Ravi Shankar, A Class of Hypercube-like Networks," Proc. of the 5th Symp. on Parallel and Distributed Processing, pp.800-803, (1993). {3