# The Study of Vertex Fault-tolerance for Multiple Spanning Paths in Hypercube


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**ABSTRACT**

This thesis is a discussion of nature about study of vertex fault-tolerance for multiple spanning paths in n-dimensional hypercube. Let be a family of G = (Vb∪Vw, E) where Kb (?} Vb) ∪ Kw (?} Vw) = {si, ti | 1 ?T i ?T (|Kb| + |Kw|) / 2} is the set of fault-free vertices, Fb ?} Vb and Fw ?} Vw are sets of faulty vertices. The family is balanced if |Kw| + 2|Fw| = |Kb| + 2|Fb|. The family connectable if there exist (|Kb| + |Kw|) / 2 spanning paths P(si, ti), for 1 ?T i ?T (|Kb| + |Kw|) / 2, in G – Fb – Fw. We show that every balanced family of hypercube Qn is connectable if |Fb| + |Fw| + |Kb| + |Kw| + |Fe| ?T n, 4|Fb| + 2|Kb| + |Fe| ?T n + 1 and 4|Fw| + 2|Kw| + |Fe| ?T n + 1, for n ?d 3. Applying this result, we can construct the fault-free cycles with length 2n – 2fmax in Qn – Fv – Fe, for fmax = max{|Fw|, |Fb|} ?T – 1, |F?e| ?T n – 1 – 4fmax. We can also construct the fault-free paths of length 2n – 2fmax – 1 (2n – 2fmax) between every pair of vertices of different (same) set in Qn – Fv – Fe, fmax = max{|Fw|, |Fb|} ?T – 1, |F?e| ?T n – 1 – 4fmax (fmax = max{|Fv∩Vj|, |Fv∩Vi| + 1} ?T for i, j = {b,w}, |F?e| ?T n + 1 – 4fmax). Applying these results, we can obtain some vertex fault-tolerant Hamiltonian properties for hypercube networks. We can obtain that Qn – Fv – Fe is a Hamiltonian and Hamiltonian laceable graph for |Fb| = |Fw| ?T – 1. We will further investigate more related vertex fault-tolerant Hamiltonian properties of more bipartite interconnection networks.



Keywords : n-dimensional hypercube ; balanced family ; connectable ; Hamiltonian laceable graph


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