

# 貝茲曲線的次數縮減：一個放蕊步驟

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## 摘要

當雲形曲線(B-spline)在不同系統之間轉換時，藉由次數提升(Degree Raising)及次數縮減(Degree Reduction)來轉換雲形曲線的次數是很常見的。本研究的主要目的在於使用放蕊步驟(Blossoming Approach)對貝茲曲線(Bezier Curve)的次數縮減作進一步的探討。本研究提出一個次數縮減的演算法使得能夠找出次數縮減後貝茲曲線的控制點(Control Points)。此演算法比舊往的演算法更容易瞭解與程式化並且能夠表達出控制點的幾何關係。此外，本研究提出矩陣的運算於放蕊演算法來改良放蕊演算法且利用此改良演算法來建構貝茲曲線。

關鍵詞：次數縮減；次數提升；放蕊步驟

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