

# An Upper Bound for the Circular Chromatic Number of Mycielski Graphs

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## ABSTRACT

In a search for triangle-free graphs with arbitrarily large chromatic numbers, Mycielski ([15]) developed a graph transformation that transforms a graph  $G$  into a new graph  $M(G)$ , we now call the Mycielskian of  $G$ . For  $t \geq 2$ ,  $Mt(G) = M(Mt-1(G))$ . The problem of determining the circular chromatic numbers of these graphs has been investigated in many papers. In this thesis, we shall study the range of  $\chi_c(Mt(G))$ , especially when  $G$  is a complete graph  $(K_n)$  or a circular complete graph  $K_{k,d}$ . In [3], Chang, Huang, and Zhu proved that if  $\chi_c(G) \leq X(G) - r$  with  $r = 1/2$  or  $1/3$ , then  $\chi_c(Mt(G)) \leq (M(G) - r)$  for every positive integer  $t$ . We find that this property is also true for  $r = 2/3$ . That is, when  $\chi_c(G)$  is close to  $X(G) - 1$ ,  $\chi_c(Mt(G))$  will also be close to  $\chi_c(M(G)) - 1$  for every positive integer  $t$ .

Keywords : Mycielski graph, circular chromatic number

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