

# Adjacent Vertices Fault-tolerance Hamiltonian Laceability of Hypercube

張佾驥、洪春男

E-mail: 9501028@mail.dyu.edu.tw

## ABSTRACT

Let  $Q_n$  be  $n$ -dimensional hypercube. In this thesis, we show that  $Q_n - F$  is Hamiltonian laceable where  $F$  is the set of  $f \leq (n-3)$  pairs of adjacent faulty vertices and  $(n-2-f)$  faulty edges. We also show that  $Q_n - F$  is hyper-Hamiltonian laceable where  $F$  is the set of  $f \leq (n-3)$  pairs of adjacent faulty vertices and  $(n-3-f)$  faulty edges. Applying these results, we can construct the fault-free path between  $s, t$  with at least  $2n - 2f' + 1$  for  $s, t$  are different color and  $2n - 2f'$  for  $s, t$  are the same color in  $Q_n - F'$  where  $2 \leq |F'| = f' \leq (n-2)$  and  $F'$  is the faulty vertices set contains at least a black vertex and a white vertex. And we can construct the fault-free cycle with at least  $2n - 2f' + 2$  in  $Q_n - F'$  where  $2 \leq |F'| = f' \leq (n-1)$  and  $F'$  is the faulty vertices set contains at least a black vertex and a white vertex. The best result thus far is the length  $2n - 2f'$ .

Keywords :  $n$ -dimensional hypercube, Hamiltonian laceable, hyper-Hamiltonian laceable.

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