

# Maximum Stiffness Design and Minimum Weight Design of Laminated Composite Plates with Meta-Heuristic Algorithms

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## ABSTRACT

The displacement and first-ply failure load of laminated composite plates are calculated by finite element method based on layerwise linear displacement theory. The analysis results have differences slightly with published practical results. The research uses a genetic algorithm, ant colony optimization and Hybrid-ANT to find the optimal stacking sequence and thickness of laminated composite plates under various loading, aspect ratios, thicknesses and boundary conditions. The maximum stiffness design of laminated composite plates with based on present analysis method subject to constant thicknesses and unconstant thicknesses. The minimum weight design of laminated composite plates with based on first-ply failure analysis technique subject to strength and thicknesses constrains. The objective of the present research is to explore various techniques for improving the efficiency of the optimal algorithm. The optimal fiber angles and thicknesses are solved by three type optimal algorithms.

Keywords : Genetic Algorithm ; Ant Colony Optimization ; Hybrid-ANT ; Finite Element Method ; Layerwise Linear Displacement Theory ; Minimum Weight Design

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