

# Vibration analysis of a cracked simply supported beam with a moving load of constant speed

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## ABSTRACT

In this study, a hybrid numerical/analytical method that permit the efficient calculation of dynamic characteristics of a cracked simply supported beam with a moving load of constant speed is presented. First, assuming the beam obeying the Euler-Bernoulli beam theory, the equation of motion of the system is derived. By using the transfer matrix method, eigensolutions (natural frequencies and mode shapes) of the cracked system can be determined. Forced response of the moving load can be obtained by model expansion theory. Some numerical results are calculated and compared with the previous researches.

Keywords : Euler-Bernoulli, transfer matrix, eigensolutions, model expansion theory

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