

Analysis of Composite Laminates Using Interlaminar Stress Continuity Theory with Interfacial Slip

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ABSTRACT

The composites laminates suffer mostly on the low strength in the thickness direction upon loading. The delamination failure inside the laminate usually occurs on these weak interfaces. Therefore, the interlaminar stresses deserve more attentions in the analysis of composite laminates. This research focuses on developing an interlaminar stress continuity theory to simulate the effect of interfacial slippage on the resonance frequency and modal stress distribution of the composite laminate. In addition, the influences of the slip constant, location and area of the interfacial slip on the dynamic characteristics of the laminate are also studied. This sensitivity study provides a feasibility evaluation for using the modal properties in the non-destructive testing of the defected laminates. Since the derived displacement field of the theory has satisfied the interfacial shear stress continuity, the interlaminar shear stress components can be calculated directly from the constitutive equations. In this thesis, the resonance frequencies and the associated modal shapes of the laminate with different slip constants, stacking sequences and locations of interfacial slip are investigated. The results of the theory are justified by several numerical examples. It is found that the interfacial defect has more influence on the higher modes than the lower ones, whereas the resonance frequency suffers the most lowering as the defect happens to be in the nodal line of the v_{ii} corresponding mode.

Keywords : Interlaminar stress continuity theory, Interlaminar shear stresses, Linear interfacial slip

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