

# 具層間滑移之層間應力連續理論的積層版分析

張石平、李春穎 陳志鏗

E-mail: 9419575@mail.dyu.edu.tw

## 摘要

由於複合材料積層板在厚度方向上強度較差，尤其是在層間介面上。此一強度較弱之介面，常於受力時形成脫層破壞。因此，積層板在層與層間的層間應力，一直是設計分析時最被關注之焦點。本研究即針對此一問題，應用層間應力連續理論，探討積層板在可能存在介面滑移之情形下，其影響各模態共振頻率、積層板層間的應力、層間滑移等現象，以及積層板的模態、頻率與瑕疵所生之位置、大小及滑移嚴重性之關係，提供一以模態參數作為積層板非破壞性檢測之方法。由於此層間應力連續理論其位移場已滿足層間剪應力連續之條件，因此所有在此之橫向層間剪應力分量皆可以直接由材料之組成律求得。本論文中以積層板在不同介面滑移常數、不同角度纖維排列、不同位置產生介面滑移下，探討共振模態頻率之改變情形。數值實例結果驗證本理論之正確性，且介面瑕疵對較高階模態之共振頻率影響較低階模態為大，而瑕疵在節點位置對該模態之共振頻率影響最大。

關鍵詞：層間應力連續理論，層間剪應力，線性介面滑移

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