

# Modal Analysis of Delaminated Composite Laminates Using Interlaminar Stress Continuity Theory

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## ABSTRACT

The strength of composite laminates is weak in the lateral direction, especially in the interlaminar interfaces. Therefore, the interlaminar stress between adjacent laminates has been the focus of the analysis of composite laminates. Delamination damage often occurs when the interlaminar interfaces are subjected to severe loading. In this study, an interlaminar stress continuity theory incorporating linear interfacial slip is used to derive a theoretical model of composite laminated structure subjected to cylindrical bending. The Hamilton's Principle is also applied to set up a finite element model. As the displacement field of generalized interlaminar stress continuity theory satisfies all the interlaminar stress continuity conditions, all three stress components can be obtained directly by the constitutive equations. The validity of the proposed theory is demonstrated first by comparing the simulation with the closed form solution. Then numerical results disclose the effects of slip constant and location of the delaminated interface on the first three modes' resonance frequencies. It is shown that the delaminated interface has higher influence on the decrement of the resonance frequency of higher order modes. In additionally the delamination at the mode of the mode shape has more pronounced effect on the reduction of that particular mode.

Keywords : Interlaminar Stress, linear interfacial slip, Interlaminar Stress Continuity Theory

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