

Vibration analysis of cracked angle-beam structure by transfer matrix method

劉錦源、林海平

E-mail: 9224880@mail.dyu.edu.tw

ABSTRACT

This investigation presents a hybrid numerical/analytical method that permit the efficient dynamic vibration characteristics of angle beam structure with transverse open cracks for used Timoshenko theory. The method utilizes a numerical implementation of a transfer matrix solution to the equation of motion. The dimensions of the eigensolution matrix will not increase with an arbitrary finite number of the angles or cracks. Therefore can be calculated the eigenvalues (natural frequency or mode shapes) of this beam structure. In this article, besides the eigenvalue problem is solved by using transfer matrix method, simultaneously demonstrated of this beam structure by experiment method.

Keywords : Transfer Matrix, Eigenvalue, Euler-Bernoulli beam, Timoshenko beam, Mode Shape

Table of Contents

第一章 緒論.....	1	1.1 研究動機.....	1	1.2 文獻回顧.....	2
1.2.1 有關Euler-Bernoulli樑理論的文獻.....	2	1.2.2 有關Timoshenko樑理論的文獻.....	5	1.3 研究方法及本文架構.....	7
1.3.1 研究方法.....	7	1.3.2 本文架構.....	8	第二章 分析方法.....	9
2.1 Euler-Bernoulli和Timoshenko理論之基本認知.....	9	2.2 各種邊界情形之介紹.....	10	2.3 Timoshenko樑之運動方程式.....	13
2.4 不同斷面之形狀係數.....	17	2.5 具彎角Timoshenko 樑結構含有破裂點 之理論分析.....	18	2.5.1 變數變換處理(Normalized)處理.....	20
2.5.2 彎角處之轉移矩陣的建立.....	21	2.5.3 破裂點處之轉移矩陣的建立.....	27	2.5.4 破裂點處之破裂形狀因子與類型.....	31
2.5.5 Fixed-Free之邊界情形與解特徵值.....	33	2.6 其它不同形式之邊界矩陣與特徵值計算.....	36	2.7 振動模態分析.....	42
第三章 結果與討論.....	44	3.1 實驗量測分析.....	44	3.1.1 實驗試體製作與材料之特性.....	45
3.1.2 實驗量測之類型.....	46	3.1.3 實驗儀器.....	48	3.1.4 實驗之安裝與架設.....	49
3.2 實驗結果與比較.....	50	3.2.1 Angle-crack理論分析計算與實驗之比較.....	51	3.2.2 Crack-angle-crack理論分析計算與實驗 之比較.....	63
第四章 結論與建議.....	67	4.1 結論.....	67	4.2 建議.....	68
參考文獻.....	69	附錄A .1 方程式(6)變數變換參數經微分處理.....	74	附錄A .2 通解方程式(11.1)、(11.2)、(11.3)經微分處理.....	74
附錄A .3 Timoshenko樑結構中彎角處轉移矩陣之內部參數.....	75	附錄A .3 Timoshenko樑結構中破裂點處轉移矩陣之內部參數.....	76		

REFERENCES

- [1] R. Roy and Jr. Craig, "Structural Dynamic: An Introducton to computer Method," 北門出版社,台北, 1985.
- [2] L. Meirovitch, "Analytical Methods in Vibrations," MacmillanCompany,London, U.K., 1967.
- [3] R.W. Clough and J. Penzien, "Dynamics of Structures," McGraw-Hill, Inc., 1975.
- [4] P.A.A Laura, J.L. Pombo and E.A. Susemihi, "A node on the vibration of clamped-free beam with mass at the free end," Journal of Sound and Vibration, 37, pp.161-168, 1974.
- [5] Y. Narkis, "Identification of crack location in vibrating simply- supported beams," Journal of sound and vibration, 172(4), pp.549-558 ,1994.
- [6] Y. Narkis and E. Elmalah, "Crack identification in a cantileverbeam under uncertain end condition," Journal of mechanics and sciences, 38(5), pp.499-507, 1996.
- [7] S.H. Farghaly, Comment and future results on "Analysis of the effect of cracks on the natural frequency of a cantilever beam," Journal of sound and vibration, 169(5), pp.704-708, 1994.
- [8] S. Masoud, M.A. Jarrah and M. Al-Maamory, "Effect of crack depth on the natural frequency of a prestressed fixed-fixed beam," Journal of sound and vibration, 214(2), pp.201-212, 1998.
- [9] M. Boltezar, B. Strancar and A. Kuhelj, "Identification of trans- verse crack location in flexural vibration of free-free beam," Journal of sound

and vibration, 211(5), pp.729-734, 1998.

- [10] A.P. Bovsunovsky and V.V. Matveev, "Analytical approach to the determination of dynamic characteristics of a beam with a losing crack," *Journal of Sound and Vibration*, 235(3), pp.415-434, 2000.
- [11] Y. Bammios, E. Douka and A. Trochidis, "Crack identification in beam structures using FEM," *Journal of Sound and Vibration*, 256(2), pp.287-297, 2002.
- [12] J.K. Sinha, M.I. Friswell and S. Edwards, "Simplified models for the location of cracks in beam structures using measured," *Journal of sound and vibration*, 251(1), pp.13-38, 2002.
- [13] H.P. Lin and C.K. Chen, "Analysis of cracked beam by transfer matrix method," *The 25th national conference on theoretical and applied mechanics*, 2001.
- [14] H.P. Lin, S.C. Chang and J.D. Wu, "Beam vibration with an arbitrary number of cracks," *Journal of sound and vibration*, 258(5), pp.987-999, 2002.
- [15] H.P. Lin and J. Ro, "Vibration analysis of planar serial-frame structures," *Journal of sound and vibration*, 262, pp.1113-1131, 2003.
- [16] M.H.F Dado and O. Abuzeid, "Coupled transverse and axial vibratory behavior of cracked beam with end mass and rotary inertia," *Journal of sound and vibration*, 261, pp.675-696, 2003.
- [17] S.P. Timoshenko, D.H. Young and W.Jr. Weaver, "Vibration problems in Engineering, 4th Ed" John Wiley & Sons Inc., 1974.
- [18] R.W. Traill-Nash and A. R. Collar, "The effects of shear flexibility and rotatory inertia on the bending vibrations of beams," *Journal of mechanics and applied mathematics*, 6, pp.186-213, 1953.
- [19] G.R. Cowper, "The shear coefficient in Timoshenko beam theory," *Journal of applied mechanics*, pp.335-340, 1966.
- [20] D.A. Grant, "The effect of rotary inertia and shear deformation on the frequency and normal mode equations of uniform beams carrying a concentrated mass," *Journal of Sound and Vibration*, 57(3), pp.357-365, 1978.
- [21] A.M. Horr and L.C. Schmidt, "Close-form solution for the Timoshenko beam theory using a computer-based mathematical package," *Journal of computer and structure*, 55(3), pp.405-412, 1995.
- [22] H. Abramovich, "A note on experimental investigation on a vibrating Timoshenko cantilever beam," *Journal of sound and vibration*, 160(1), pp.167-171, 1993.
- [23] S.H. Farchaly and M.G. Shebl, "Exact frequency and mode shape formulae for studying vibration and stability of Timoshenko beam system," *Journal of sound and vibration*, 180, pp.205-227, 1995.
- [24] T.C. Tsai and Y.Z. Wang, "Vibration analysis and diagnosis of a cracked shaft," *Journal of sound and vibration*, 192(3), pp.607-620, 1996.
- [25] B. Geist and J.R. McLaughlin, "Eigenvalue formulas for the uniform Timoshenko beam: the free-free problem," *Electronic research announcements of the American mathematical society*, 4, pp.12-17, 1998.
- [26] M. Kisa, J. Brandon and M. Topcu, "Free vibration analysis of cracked beams by a combination of finite elements and component mode synthesis methods," *Computer and Structures*, 67, pp.215-223, 1998.
- [27] S.M. Han, H. Benaroya and T. Wei, "Dynamics of transversely vibrating beams using four engineering theories," *Journal of sound and vibration*, 225(5), pp.935-988, 1999.
- [28] S.P. Lele and S.K. Maiti, "Modeling of transverse vibration of short beams for crack detection and measurement of crack extension," *Journal of sound and vibration*, 257(3), pp.559-583, 2002.
- [29] N.G. Stephen, "A check on the accuracy Timoshenko's beam theory," *Journal of sound and vibration*, 257(4), pp.809-812, 2002.
- [30] W. Taylor and C.B. Yau, "Boundary control of a rotating Timoshenko beam," *Journal of ANZIAM*, 44(E), pp.E143-E184, 2003.
- [31] S.M. Han, H. Benaroya and T. Wei, "Dynamics of transversely vibrating beams using four engineering theories," *Journal of sound and vibration*, 225(5), pp.935-988, 1999.
- [32] A.N. Kounadis, "One the derivation of equation of motion of a vibrating Timoshenko column," *Journal of sound and vibration*, 73(2), pp.177-184, 1980.
- [33] 吳嘉慶, "剛架的振動分析," 碩士論文, 國立中興大學, 2001.
- [34] G.B. Warburton, "The dynamical behavior of structures," 紅橋書局, 台北.
- [35] W.M. Ostachwicz, "Decision of crack stiffness and flexibility," *Journal of sound and vibration*, 1991.
- [36] T.G. Chondros and A.D. Dimarogonas, "Identification of cracks in welded joints of complex structures," *Journal of sound and vibration*, 69(4), pp.531-538, 1980.
- [37] S. Masoud, M.A. Jarrah and M. Al-Maamory, "Effect of crack depth on the natural frequency of a prestressed fixed-fixed beams," *Journal of sound and vibration*, 214(2), pp.201-212, 1998.
- [38] J.H. Lau, "Fundamental frequency of a constrained beam," *Journal of Sound and Vibration*, 78(1), pp.154-157, 1981.
- [39] D. J. Ewins, "Modal testing: theory and practices," Research studies press Ltd., England, 1986.
- [40] 陳宏謀, "結構學觀念分析 上冊," 浩瀚系列叢書, 台北, 1992.
- [41] 左利時, "結構學 下冊," 三民書局, 台北, 1988.
- [42] 徐耀賜, "梁結構," 全華科技圖書股份有限公司, 台北, 2001.

[43] Aslam Kassimali 原著, 華根、李光台、宋裕棋 譯著, "結構分析," 六合出版社, 台北, 1997.