

# 用轉移矩陣法做破壞樑結構之振動分析與研究

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## 摘要

本研究係利用結合理論數值分析之方法，其目的在探討Timoshenko樑之理論應用於具彎角之樑結構中含有破裂點時之動態特性。由導出的運動方程式，利用轉移矩陣法(transfer matrix method)，其特徵值解之矩陣階數不會隨著彎角或破裂點之數目增加而增加，由此可以計算出此樑結構系統之特徵解即自然頻率及振型。本文除了探討分析理論計算出的特徵值外，同時藉由實驗方法對此理論模式加以驗證。

關鍵詞：轉移矩陣、特徵值、Euler-Bernoulli、Timoshenko、振態

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## 參考文獻

- [1] R. Roy and Jr. Craig, "Structural Dynamic: An Introducton to computer Method," 北門出版社,台北, 1985.
- [2] L. Meirovitch, "Analytical Methods in Vibrations," MacmillanCompany,London, U.K., 1967.
- [3] R.W. Clough and J. Penzien, "Dynamics of Structures," McGraw-Hill, Inc., 1975.
- [4] P.A.A Laura, J.L. Pombo and E.A. Susemihi, "A node on the vibration of clamped-free beam with mass at the free end," Journal of Sound and Vibration, 37, pp.161-168, 1974.
- [5] Y. Narkis, "Identification of crack location in vibrating simply- supported beams," Journal of sound and vibration, 172(4), pp.549-558, 1994.
- [6] Y. Narkis and E. Elmalah, "Crack identification in a cantileverbeam under uncertain end condition," Journal of mechanics and sciences, 38(5), pp.499-507, 1996.
- [7] S.H. Farghaly, Comment and future results on "Analysis of the effect of cracks on the natural frequency of a cantilever beam," Journal of sound and vibration, 169(5), pp.704-708, 1994.
- [8] S. Masoud, M.A. Jarrah and M. Al-Maamory, "Effect of crack depth on the natural frequency of a prestressed fixed-fixed beam," Journal of sound and vibration, 214(2), pp.201-212, 1998.
- [9] M. Boltezar, B. Strancar and A. Kuhelj, "Identification of trans- verse crack location in flexural vibration of free-free beam," Journal of sound and vibration, 211(5), pp.729-734, 1998.
- [10] A.P. Bovsunovsky and V.V. Matveev, "Analytical approach to the determination of dynamic charcteristics of a beam with a losing crack,"

Journal of Sound and Vibration, 235(3), pp.415-434, 2000.

- [11] Y. Bamnios, E. Douka and A. Trochidis, "Crack identification in beam structures using FEM," Journal of Sound and Vibration, 256(2), pp.287-297, 2002.
- [12] J.K. Sinha, M.I. Friswell and S. Edwards, "Simplified models for the location of cracks in beam structures using measured," Journal of sound and vibration, 251(1), pp.13-38, 2002.
- [13] H.P. Lin and C.K. Chen, "Analysis of cracked beam by transfer matrix method," The 25th national conference on theoretical and applied mechanics, 2001.
- [14] H.P. Lin, S.C. Chang and J.D. Wu, "Beam vibration with an arbitrary number of cracks," Journal of sound and vibration, 258(5), pp.987-999, 2002.
- [15] H.P. Lin and J. Ro, "Vibration analysis of planar serial-frame structures," Journal of sound and vibration, 262, pp.1113-1131, 2003.
- [16] M.H.F Dado and O. Abuzeid, "Coupled transverse and axial vibratory behavior of cracked beam with end mass and rotary inertia," Journal of sound and vibration, 261, pp.675-696, 2003.
- [17] S.P. Timoshenko, D.H. Young and W.Jr. Weaver, "Vibration problems in Engineering, 4th Ed" John Wiley & Sons Inc., 1974.
- [18] R.W. Traill-Nash and A. R. Collar, "The effects of shear flexibility and rotatory inertia on the bending vibrations of beams," Journal of mechanics and applied mathematics, 6, pp.186-213, 1953.
- [19] G.R. Cowper, "The shear coefficient in Timoshenko beam theory," Journal of applied mechanics, pp.335-340, 1966.
- [20] D.A. Grant, "The effect of rotary inertia and shear deformation on the frequency and normal mode equations of uniform beams carrying a concentrated mass," Journal of Sound and Vibration, 57(3), pp.357-365, 1978.
- [21] A.M. Horr and L.C. Schmidt, "Closed-form solution for the Timoshenko beam theory using a computer-based mathematical package," Journal of computer and structure, 55(3), pp.405-412, 1995.
- [22] H. Abramovich, "A note on experimental investigation on a vibrating Timoshenko cantilever beam," Journal of sound and vibration, 160(1), pp.167-171, 1993.
- [23] S.H. Farchaly and M.G. Shebl, "Exact frequency and mode shape formulae for studying vibration and stability of Timoshenko beam system," Journal of sound and vibration, 180, pp.205-227, 1995.
- [24] T.C. Tsai and Y.Z. Wang, "Vibration analysis and diagnosis of a cracked shaft," Journal of sound and vibration, 192(3), pp.607-620, 1996.
- [25] B. Geist and J.R. McLaughlin, "Eigenvalue formulas for the uniform Timoshenko beam: the free-free problem," Electronic research announcements of the American mathematical society, 4, pp.12-17, 1998.
- [26] M. Kisa, J. Brandon and M. Topcu, "Free vibration analysis of cracked beams by a combination of finite elements and component mode synthesis methods," Computer and Structures, 67, pp.215-223, 1998.
- [27] S.M. Han, H. Benaroya and T. Wei, "Dynamics of transversely vibrating beams using four engineering theories," Journal of sound and vibration, 225(5), pp.935-988, 1999.
- [28] S.P. Lele and S.K. Maiti, "Modeling of transverse vibration of short beams for crack detection and measurement of crack extension," Journal of sound and vibration, 257(3), pp.559-583, 2002.
- [29] N.G. Stephen, "A check on the accuracy of Timoshenko's beam theory," Journal of sound and vibration, 257(4), pp.809-812, 2002.
- [30] W. Taylor and C.B. Yau, "Boundary control of a rotating Timoshenko beam," Journal of ANZIAM, 44(E), pp.E143-E184, 2003.
- [31] S.M. Han, H. Benaroya and T. Wei, "Dynamics of transversely vibrating beams using four engineering theories," Journal of sound and vibration, 225(5), pp.935-988, 1999.
- [32] A.N. Kounadis, "On the derivation of equation of motion of a vibrating Timoshenko column," Journal of sound and vibration, 73(2), pp.177-184, 1980.
- [33] 吳嘉慶, "剛架的振動分析," 碩士論文, 國立中興大學, 2001.
- [34] G.B. Warburton, "The dynamical behavior of structures," 紅橋書局, 台北.
- [35] W.M. Ostachwicz, "Decision of crack stiffness and flexibility," Journal of sound and vibration, 1991.
- [36] T.G. Chondros and A.D. Dimarogonas, "Identification of cracks in welded joints of complex structures," Journal of sound and vibration, 69(4), pp.531-538, 1980.
- [37] S. Masoud, M.A. Jarrah and M. Al-Maamory, "Effect of crack depth on the natural frequency of a prestressed fixed-fixed beams," Journal of sound and vibration, 214(2), pp.201-212, 1998.
- [38] J.H. Lau, "Fundamental frequency of a constrained beam," Journal of Sound and Vibration, 78(1), pp.154-157, 1981.
- [39] D. J. Ewins, "Modal testing: theory and practices," Research studies press Ltd., England, 1986.
- [40] 陳宏謀, "結構學觀念分析 上冊," 浩瀚系列叢書, 台北, 1992.
- [41] 左利時, "結構學 下冊," 三民書局, 台北, 1988.
- [42] 徐耀賜, "梁結構," 全華科技圖書股份有限公司, 台北, 2001.
- [43] Aslam Kassimali 原著, 華根、李光台、宋裕棋 譯著, "結構分析," 六合出版社, 台北, 1997.