# Three-Dimensional Stress Analysis of Composite Laminates Using Interlaminar Stress Continuity Theory


## 冀彪、李春穎

E-mail: 9224314@mail.dyu.edu.tw



**ABSTRACT**

The composite laminates suffer mostly on the low strength in the thickness direction upon loading. Therefore, the interlaminar stresses deserve more attentions in the analysis of composite laminates, especially the stresses at the free edge or corner. In this study, an interlaminar stress continuity theory is adopted to the analysis of the interlaminar stresses. The derived finite model using Hamilton's Principle can be used to calculate all six stress components by employing the constitutive equations. With this viable tool, the influences of dimensional size, stacking sequence and material property on the interlaminar stresses at the material interfaces, the free edges of the laminate are studied. With the simulation results of different laminates, we found the magnitude of the interlaminar stresses at the free edge increases with the width of the specimen and the heterogeneity of the bending rigidity through the thickness.



Keywords : interlaminar stress continuity theory, interlaminar stresses, free-edge stresses


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