層用應力連續理論用於積層板之三維應力分析

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摘 要

由於複材積層板在厚度方向上強度較差，因此，積層板在厚度方向上之橫向應力，尤其是層與層間的層間應力，一直是設計分析時最關注之焦點。而此層間應力又在結構所謂之自由邊或自由邊角更顯現其破壞性。本研究即針對此一問題，應用層間應力連續理論，推導結構在受力或控制變形下之理論模式，並用漢米爾頓原理推導建立其有限元素模型。由於層間應力連續理論其位移場已滿足層間位移及應力連續之條件，因此所有六個應力分量皆可以直接由材料之組成律求得。應用此有限元模型為工具，本研究將探討受力、控制變形、堆疊順序、材料性質對積層板結構內部異質介面應力、自由邊角層間應力之影響，並期能為緩和自由邊角之層間應力，增加結構之負載能力，提供設計上之方向。由數值實例之分析結果驗證，彎曲時試片的寬度越大，自由邊上之橫向應力會隨之增加，而厚度方向上之剛度均質性越好，則可緩和自由邊之橫向層間應力。

關鍵詞 : 層間應力連續理論、層間應力、自由邊效應

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