

# 適用無線環境之橢圓曲線加速演算法研究

周智禾、曹偉駿

E-mail: 9222446@mail.dyu.edu.tw

## 摘要

在1985年，Koblitz和Miller提出了橢圓曲線密碼系統，由於其所使用的金鑰大小可以大幅降低的特性，因此隨即受到了廣泛的重視。橢圓曲線密碼系統160位元金鑰長度的安全性，同等於RSA公開金鑰密碼系統的1024位元金鑰長度，故非常適合運用在具較小的記憶體與運算能力的裝置上，如智慧卡。IEEE、ANSI和ISO等都已經將橢圓曲線列為其標準，因此，我們預期它將是未來各種公開金鑰密碼系統的主流之一。橢圓曲線密碼系統的效率依賴在純數積運算上，因此，基於效率上的考量，發展出其加速演算法是必要的。在1994年，Lim和Lee提出一個用在無線環境且較具彈性的方法，有效的利用事先計算來加快指數運算的速度，其方法亦可使用在加速橢圓曲線的純數積運算上，我們稱之為LLECC方法。然而，LLECC方法使用在記憶體空間越小的裝置上，會越沒有效率，因此，在本篇論文中，我們將提出一個比LLECC方法更有效率的加速演算法。首先，我們將修改LLECC方法的演算法以降低事先運算所需花費的儲存空間，並進一步結合符號位元表示法及Multidoubling，以提出一套有效率的加速演算法，同時我們的方法亦可應用在加速橢圓曲線多點的乘法運算上。相較於LLECC方法，我們的方法不但降低了記憶體空間儲存量，並進一步地改善其運算效率，故本研究所提出之加速演算法較適用於無線的環境之下。根據我們的模擬結果，在有限場及仿射座標系下之160位元的橢圓曲線密碼系統中，本方法可以減少11%的計算複雜度以及降低記憶體空間達21%。最後，在實作方面，我們將利用本研究所提出之演算法，來加速橢圓曲線數位簽章系統中的純數積運算於個人數位助理中。

關鍵詞：公開金鑰密碼系統、橢圓曲線密碼系統、純數積運算、點乘法、多點乘法。

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## 參考文獻

- [1] ANSI X9.31, Digital Signatures using Reversible Public Key Cryptography for the Financial Services Industry (rDSA), 1998.
- [2] ANSI X9.62, Public Key Cryptography for the Financial Services Industry: the Elliptic Curve Digital Signature Algorithm (ECDSA), 1999.
- [3] ANSI X9.63, Public Key Cryptography for the Financial Services Industry: Elliptic Curve Key Agreement and Key Transport Protocols, Working Draft, 2000.
- [4] I. F. Blake, G. Seroussi and N. P. Smart, Elliptic Curves in Cryptography, The Press Syndicate of the University of Cambridge, 1999.
- [5] D. Boneh and M. Franklin, "Identity-Based Encryption from the Weil Pairing," Advances in Cryptology-Crypto'2001, Springer-Verlag, pp. 213-229, 2001.

- [6] J. Bos and M. Coster, "Addition Chain Heuristics," *Advances in Cryptology-Crypto'89*, Springer-Verlag, Vol. 415, pp. 400-407, 1990.
- [7] E. F. Brickell, D. M. Gordon, K. S. McCurley and D. Wilson, "Fast Exponentiation with Precomputation," *Advances in Cryptology-Eurocrypt'92*, Springer-Verlag, pp. 200-207, 1992.
- [8] Certicom Corporation, URL: <http://www.certicom.com/>.
- [9] H. Cohen, *A Course in Computational Algebraic Number Theory*, Graduate Texts in Mathematics, Springer-Verlag, 1993.
- [10] W. Diffie and M.E. Hellman, "New Directions in Cryptography," *IEEE Transactions in Information Theory*, Vol. IT-22, pp. 644-654, Nov. 1976.
- [11] P. Downey, B. Leony and R. Sethi, "Computing Sequences with Addition Chains," *SIAM Journal of Computing*, pp. 638-696, 1981.
- [12] T. ElGamal, "A Public-Key Cryptosystem and a Signature Scheme Based on Discrete Logarithms," *IEEE Transactions on Information Theory*, Vol. IT-31, No. 4, pp. 469-472, 1985.
- [13] FIPS 186-2, National Institute of Standards and Technology, Digital Signature Standard, FIPS Publication 186-2, Available from <http://csrc.nist.gov/encryption/>, 2000.
- [14] D. M. Gordon, "A Survey of Fast Exponentiation Methods," *Journal of Algorithms*, Vol. 27, pp. 129-146, 1998.
- [15] J. Guajardo and C. Paar, "Efficient Algorithms for Elliptic Curve Cryptosystems," *Advances in Cryptology-Crypto'97*, LNCS, Springer-Verlag, Vol. 1294, pp. 342-356, 1997.
- [16] Y. Han and P. C. Tan, "Direct Computation for Elliptic Curve Cryptosystems," *Pre-proc. Cryptographic Hardware and Embedded Systems (CHES)'99*, Springer-Verlag, pp. 328-340, 1999.
- [17] IEEE P1363, Standard Specifications for Public-Key Cryptography, <http://grouper.ieee.org/groups/1363/index.html>, 2000.
- [18] ISO/IEC 14888-3, Information Technology-Security Techniques- Digital Signature with Appendix-Part 3: Certificate-based Mechanisms.
- [19] ISO/IEC 15946 series, Information Technology-Security Techniques-Cryptographic Techniques Based on Elliptic Curves, Working Draft, 1998.
- [20] ISO/IEC 9796-4, Information Technology-Security Techniques- Digital Signature with Message Recovery-Part4: Discrete Logarithm-based Mechanisms.
- [21] J. Jedwab and C. J. Mitchell, "Minimum Weight Modified Signed-Digit Representation and Fast Exponentiation," *Electronic Letter*, Vol. 25, No. 17, pp. 1171-1172, 1989.
- [22] A. Juristic and A. J. Menezes, "Elliptic Curve and Cryptography," *Dr. Dobb's Journal*, pp. 26-35, 1997.
- [23] D. E. Knuth, "Seminumerical Algorithms 2nd," *The Art of Computer Programming*, Vol. 2, Addison-Wesley, 1983.
- [24] N. Koblitz, "Elliptic Curve Cryptosystems," *Math. Comp.*, Vol. 48, No. 17, pp. 203-209, 1987.
- [25] N. Koblitz, "Constructing Elliptic Curve Cryptosystems in Characteristic 2," *Crypto'90*, pp. 156-167, 1990.
- [26] C. S. Lai and W. C. Kuo, "Speeding Up the Computations of Elliptic Curve Cryptoschemes," *International Journal of Computers & Mathematics with Applications*, Vol. 33, No. 5, pp. 29-36, March 1997.
- [27] C. H. Lim and P. J. Lee, "More Flexible Exponentiation with Precomputation," *Advances in Cryptology-Crypto'94*, Springer-Verlag, pp. 95-107, 1994.
- [28] G. W. Lo, *The Study and Implementation on Elliptic Curve Digital Signature Schemes*, Master Thesis, NCKU, Taiwan, 2000.
- [29] A. J. Menezes and S. A. Vanstone, "Elliptic Curve Cryptosystems and Their Implementation," *Journal of Cryptology*, Vol. 6, No. 4, pp. 209-224, 1993.
- [30] A. J. Menezes, T. Okamoto and S. A. Vanstone, "Reducing Elliptic Curve Logarithms to a Finite Field," *IEEE Transactions on Information Theory*, pp.1639-1646, 1993.
- [31] V. Miller, "Uses of Elliptic Curves in Cryptography," *Advances in Cryptology-Crypto'85*, Springer-Verlag, pp. 417-426, 1985.
- [32] A. Miyaji, T. Ono and H. Cohen, "Efficient Elliptic Curve Exponentiation (I)," *IEICE Technical Report*, ISEC97-16, 1997.
- [33] F. Morain and J. Olivos, "Speeding Up the Computations on an Elliptic Curve Using Addition-Subtraction Chains," *Info. Theory Appl.*, pp. 531-543, 1990.
- [34] V. Muller, "Efficient Algorithms for Multiplication on Elliptic Curves," *Proc. GI-Arbeitskonferenz Chipkarten 1998*, TU Munchen, 1998.
- [35] National Institute of Standards and Technology, "Digital Signature Standard," *Communications of the ACM*, Vol. 35, No. 7, pp. 36-40, July 1992.
- [36] A. M. Odlyzko, "Discrete Logs in a Finite Field and Their Cryptographic Significance," *Advances in Cryptology-Eurocrypt '84*, Springer-Verlag, pp.224-314, 1985.
- [37] P. Oorschot Van and M. Wiener, "Parallel Collision Search with Cryptanalytic Applications," *Journal of Cryptology*, pp. 1-28, 1999.
- [38] J. Pollard, "Monte Carlo Methods for Index Computation," *Math. Comp.*, pp. 918-924, 1978.
- [39] S. Pohlig and M. Hellman, "An Improved Algorithm for Computing Logarithms over and Its Cryptographic Significance," *IEEE Transactions on Information Theory*, pp.106-110, 1978.
- [40] G. Poupard and J. Stern, "A Practical and Provable Secure Design of on the Fly Authentication and Signature Generation," *Advances in Cryptology- proceedings of Eurocrypt '98*, Springer-Verlag, pp.422-436, 1998.

- [41] M. O. Rabin, "Digitalized Signatures and Public-Key Functions as Intractable as Factorization," Technical Report LCS/TR212, MIT Laboratory for Computer Science, 1979.
- [42] M. O. Rabin, "Probabilistic Algorithm for Testing Primality," Journal of Number Theory, Vol. 12, pp. 128-138, 1980.
- [43] R. L. Rivest, A. Shamir and L. M. Adleman, "A Method for Obtaining Digital Signatures and Public-Key Cryptosystems," Communications of the ACM, Vol. 21, pp. 120-126, Feb. 1978.
- [44] Y. Sakai and K. Sakurai, "Efficient Scalar Multiplications on Elliptic Curves without Repeated Doublings and Their Practical Performance," Information Security and Privacy, ACISP 2000, LNCS, Springer-Verlag, Vol. 1841, pp. 59-63, 2000.
- [45] Y. Sakai and K. Sakurai, "Efficient Scalar Multiplications on Elliptic Curves with Direct Computation of Several Doublings," IEICE Transactions Fundamentals, Vol. E84-A, No. 1, pp. 120-129, 2001.
- [46] Y. Sakai and K. Sakurai, "Speeding Up Elliptic Scalar Multiplication Using Multidoubling," IEICE Transactions Fundamentals, Vol. E85-A, No. 5, pp. 1075-1083, 2002.
- [47] T. Satoh and K. Araki, "Fermat Quotients and the Polynomial Time Discrete Log Algorithm for Anomalous Elliptic Curves," Comm. Math. Univ. Sancti. Pauli., pp. 81-92, 1998.
- [48] C. P. Schnorr, "Efficient Identification and Signature for Smart Cards," Advances in Cryptology-Crypto'89, New York, Springer-Verlag, pp. 239-252, 1990.
- [49] SEC1, "Elliptic Curve Cryptography," Standards for Efficient Cryptography Group, Available from <http://www.secg.org/collateral/>, 2000.
- [50] SEC2, "Recommended Elliptic Curve Cryptography Domain Parameters", Standards for Efficient Cryptography Group, Available from <http://www.secg.org/collateral/>, 2000.
- [51] J. H. Silverman, The Arithmetic of Elliptic Curves, Graduate Texts in Mathematics 106, Springer-Verlag, New York, 1986.
- [52] J. H. Silverman, Advanced Topics in the Arithmetic of Elliptic Curves, Graduate Texts in Mathematics 151, Springer-Verlag, New York, 1994.
- [53] N. P. Smart, "The Discrete Logarithm Problem on Elliptic Curves of Trace One," Journal of Cryptology, 1999.
- [54] M. J. Wiener, "Cryptanalysis of Short RSA Secret Exponents," IEEE Transactions on Information Theory, Vol. IT-36, pp. 553-558, 1990.
- [55] E. De Win, S. Mister, B. Prenel and M. Wiener, "On the Performance of Signature based on Elliptic Curves," Algorithmic Number Theory, Proceedings Third Intern. Symb., ANTS-III, LNCS 1423, Springer-Verlag, pp. 252-266, 1998.
- [56] W. C. Yang, K. M. Lin and C. S. Lai, "A Precomputation Method for Elliptic Curve Point Multiplication," Journal of the Chinese Institute of Electrical Engineering, Vol. 9, No. 4, pp. 339-344, Nov. 2002.
- [57] S. M. Yen, C. S. Lai and A. K. Lenstra, "Multi-Exponentiation," IEE Proceedings Part-E: Computers and Digital Techniques, Vol. 141, No. 6, pp. 325-326, Nov. 1994.
- [58] S. M. Yen and C. S. Lai, "The Fast Cascade Exponentiation Algorithm and Its Application on Cryptography," Advances in Cryptology-Auscrypt'92, New York, Springer-Verlag, pp. 447-456, 1993.
- [59] 賴溪松、韓亮、張真誠, 「近代密碼學及其應用」, 松崗圖書資料公司, 民國84年9月。