

# HELE-SHAW 流場中可互溶液體運動之研究及KORTEWEG 應力的影響

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## 摘要

利用高精密度的數值方法模擬可互溶液滴，於均質的HELE-SHAW 流場中之運動現象。對影響參數，包括黏滯度差異、無因次流量（PECLET NUMBER, PE）和液滴大小以及KORTEWEG 應力，作一系統式分析，以瞭解各參數之重要性及影響效果。HELE-SHAW 流場為兩塊透明薄板，夾以極小的間隙接合而成的二維流場。由於其第三維的高度極小於其他兩維的長度，因此可將第三維的速度加以平均起來，視HELE-SHAW 流場為一非旋轉的勢流場（POTENTIAL FLOW）。計算模擬乃將統御方程式轉換為渦度-流函數之關係式，深入研究因濃度梯度所造成的速度散度，對流場之影響。當液滴黏滯度較環境流體低時，在下游部分的液滴前沿，指狀化現象受流場的控制參數影響極大，一旦黏滯度差異愈大，或是PE 值愈高時，指狀化情形將更為劇烈。至於在液滴黏滯度較環境流體高時，可溶液滴移動較慢且較為穩定，並在液滴後緣處產生一個尾巴外型。在考慮到KORTEWEG 應力的影響時，HU AND JOSEPH 在1992 年曾利用線性穩定分析結果預測負的  $\beta$  值，可避免不穩定的發生；而從我們的模擬結果亦顯示這時的液滴，前沿將因KORTEWEG 應力的影響，呈現穩定的狀態，產生相當類似KOPF-SILL AND HOMSY 實驗觀察到的尾巴形液滴（TAILED-BUBBLES）。這時的液滴外型可類比不可溶液滴對表面張力分佈的說法來預測之，且這些尾巴形液滴的移動速度均介於實驗所量測的範圍。

關鍵詞：可互溶液滴、PECLET NUMBER、KORTEWEG 應力

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