Computing Means and Covariances of Inverse-Gaussian Order Statistics for Applications in Constructing BLUEs

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ABSTRACT

WE CONSIDER THE PROBLEM OF COMPUTING MEANS, VARIANCES AND COVARIANCES FOR THE ORDER STATISTICS FROM INVERSE-GAUSSIAN DISTRIBUTIONS. INVERSE-GAUSSIAN DISTRIBUTIONS HAVE BECOME ONE OF THE COMMONLY USED STATISTICAL MODELS, PARTICULARLY USEFUL FOR MODELING THE LIFETIME FOR PRODUCTS OR DEVICES. INVERSE-GUASSIAN DISTRIBUTIONS HAVE THREE USEFUL PROPERTIES: (1) THE FAILURE FUNCTION FOR THE INVERSE GAUSSIAN IS NONMONOTONIC; IT FIRST INCREASES AND THEN DECREASES, APPROACHING A CONSTANT VALUE AS THE LIFETIME BECOMES INFINITE, (2) THE RANGE OF DISTRIBUTION SHAPES IS FAIRLY WIDE, FROM HIGHLY SKEWED TO RIGHT TO AN ALMOST NORMAL SHAPE, AND (3) THE IDEA OF FIRST PASSAGE TIME FOR AN UNDERLYING PROCESS ALLOWS APPLICATIONS TO A VARIETY OF PROCESS. ORDER STATISTICS CAN BE USED TO CONSTRUCT BEST LINEAR UNBIASED ESTIMATORS (BLUES) FOR POPULATION MEAN AND STANDARD DEVIATION. THE BLUE IS A LINEAR COMBINATION OF ORDER STATISTICS WHERE THE COEFFICIENTS ARE A FUNCTION OF MEANS, VARIANCES, AND COVARIANCES OF STANDARDIZED ORDER STATISTICS. WE PLAN TO DEVELOP COMPUTER SOFTWARE INSTEAD OF BUILDING TABLES. THE DISADVANTAGE OF TABLES ARE THAT THE TABLE HAS A LIMITATION OF NUMBER OF GIVEN VALUES, AND THAT IF THE GIVE VALUES ARE NOT LISTED IN THE TABLE, INTERPOLATION, OR WORSE, EXTRAPOLATION, HAS TO BE DONE, CAUSING ERRORS IN THE STATISTICAL MODEL. FOR CONVENIENT USAGE OF SOFTWARE, EFFICIENT ALGORITHMS ARE NEEDED. NUMERICAL METHODS CAN BE USED TO COMPUTE THE MEANS, VARIANCES AND COVARIANCES OF A SET OF STANDARDIZED INVERSE-GAUSSIAN ORDER STATISTICS SINCE MEANS AND VARIANCES ARE ONE-DIMENSIONAL INTEGRALS AND COVARIANCES ARE TWO-DIMENSIONAL. WE IMPLEMENT TWO NUMERICAL INTEGRATION METHODS: MODIFIED MIDPOINT METHODS AND GAUSSIAN QUADRATURE METHODS. EMPIRICAL RESULTS SHOW THE SHORTAGES OF THESE TWO MEHTODS: MIDPOINT METHODS ARE SLOW, EVEN SLOWER THAN SIMULATION CRUDE ESTIMATORS; GAUSSIAN QUADRATURE METHODS ARE MORE EFFICIENT BUT IT IS VERY INACCURATE WHEN THE INVERSE-GAUSSIAN SKEWNESS IS HIGH. THESE TWO SHORTAGES MOTIVATE US TO DEVELOP MONTE CARLO ESTIMATION METHODS. THE SIMULATION METHOD GENERATES M SAMPLES, EACH OF SIZE N, OF ORDER STATISTICS. THE ORDER-STATISTICS MEANS, VARIANCES, AND COVARIANCES ARE THEN ESTIMATED BY THEIR CORRESPONDING SAMPLE STATISTICS USING THE SAME M SAMPLES. FOR BETTER EFFICIENCY, WE USE CONTROL-VARIATE METHODS TO REDUCE THE VARIANCE OF CRUDE ESTIMATORS. TWO KINDS OF CONTROL VARIATES ARE DEFINED: (I) U(0,1) INTERNAL CONTROL: THE CONTROL VARIATES ARE THE ESTIMATES OF MEANS, VARIANCES, AND COVARIANCES OF A SET OF U(0,1) ORDER STATISTICS OF SIZE N. (II) EXP(1) EXTERNAL CONTROL: THE CONTROL VARIATES ARE THE ANALOGOUS MEAN, VARIANCE, AND COVARIANCE ESTIMATES OF EXP(1) ORDER STATISTICS. TO EVALUATE THE VARIANCE-REDUCTION EFFECT, SIMULATION EXPERIMENTS ARE PERFORMED. FOUR CONCLUSIONS ARE REACHED FROM SIMULATION RESULTS: (I) TWO CONTROL-VARIATE METHODS HAVE GOOD VARIANCE-REDUCTION EFFECTS; EXP(1) EXTERNAL CONTROL IS BETTER THAN U(0,1) INTERNAL CONTROL. (II) DESPITE U(0,1) OR EXP(1) CONTROL VARIATES, WHEN THE INVERSE-GAUSSIAN SKEWNESS INCREASES, THE VARIANCE-REDCUTION EFFECT FIRST INCREASES AND THEN DROPS. ALMOST NO VARIANCE REDUCTION WHEN THE SKEWNESS IS 50. (III) EXP(1) CONTROL VARIATES HAVE THE LARGEST VARIANCE REDUCTION WHEN THE INVERSE-GAUSSIAN SKEWNESS IS AROUND 2; THIS IS BECAUSE THE EXP(1) SKEWNESS IS ALSO 2. (IV) EXP(1) CAN EFFECTIVELY REDUCE VARIANCES FOR THE LARGEST ORDER N; THIS IS BECAUSE THE EXP(1) DISTRIBUTION HAS A LONG RIGHT TAIL AS WELL AS THE INVERSE-GAUSSIAN DISTRIBUTION. U(0,1) IS A BOUNDED DISTRIBUTION AND HENCE HAS LITTLE VARIANCE-REDUCTION EFFECT FOR THE LARGEST ORDER. SINCE THE VARAINCES AT THE LARGEST ORDER ARE USUALLY HIGHER THAN OTHER ORDERS, EXP(1) CONTROL VARAITES HAS BETTER VARIANCE

REDUCTION THAN U(0,1) CONTROL VARIATES.

Keywords : BEST LINEAR UNBIASED ESTIMATOR, CHI-SQUARE METHOD, CONTROL VARIATES, GAUSSIAN QUADRATURE METHOD, INVERSE GAUSSIAN DISTRIBUTION, INVERSE TRANSFORMATION, MIDPOINT METHOD, MONTE CARLO ESTIMATION, NUMERICAL INTEGRATION, ORDER STATISTICS, VARIANCE REDUCTION

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