

The Study of Rational Function BCH and Reed-Solomon Codes

阮仲義、胡大湘

E-mail: 381834@mail.dyu.edu.tw

ABSTRACT

Due to the development of digital technology, the demands of data transmission and connecting people are improved more and more. Communication systems are researched and developed to serve for higher demands not only speed but also quality. Error control coding is one of related research fields that improve quality for information systems. In this work, a new bound on the minimum distance of non-Bose-Chaudhuri-Hocquenghem (BCH) codes and Reed-Solomon (RS) codes based on “Rational function” is presented. This bound improve upon the BCH bound, and for some case upon the Hartmann-Tzeng (HT) bound. The main research’s purpose is to improve the capability of correcting errors and erasures based on the new bound. Both modified Berlekamp Algorithm (BA) and Euclidean Algorithm (EA) are presented to perform all necessary decoding steps.

Keywords : Non-BCH codes、RS codes、Rational function、BCH bound、HT bound、Berlekamp algorithm、Euclidean algorithm

Table of Contents

中文摘要.....	iii	ABSTRACT.....	iv	TABLE OF CONTENTS.....	v	LIST OF FIGURES.....	vii	LIST OF TABLES.....	viii
Chapter I. Introduction and Motivation.....	1	1.1 Communication System.....	1	1.2 Role of Coding in Communication System.....	3	1.2.1 Channel coding (Error control coding).....	3	1.2.2 Block codes.....	4
1.3 Motivation of Study.....	7	Chapter II. Literatures Review.....	9	2.1 q-ary linear block codes.....	9	2.2 Non-binary BCH Codes.....	10	2.3 Reed-Solomon Codes.....	12
2.3.1 Encoding of Reed-Solomon codes.....	13	2.3.1.1 Non-systematic form.....	13	2.3.1.2 Systematic form.....	14	2.3.2 Decoding of Reed-Solomon Codes.....	17	2.3.2.1 The Berlekamp Algorithm	17
2.3.2.2 The Euclidean Algorithm(EA)	21	2.4 Some concept of bounds on the minimum distance [5].....	22	2.4.1 BCH bound:.....	23	2.4.2 Hartmann-Tzeng (HT) bound:.....	23	2.5 Correction of errors and erasures.....	23
2.6 The “Ration Functions” [3],[4] method makes a new bound on the minimum distance.....	27	Chapter III. “Rational functions” method applied to Reed-Solomon Codes.....	30	3.1 Reed-Solomon codes with “Rational functions” method.....	30	3.2 Decoding of the new method [1], [2], [8].....	34	3.3 Find error evaluation for error and erasure symbols:.....	36
3.4 Correction of errors and erasures with “Rational functions” method.....	38	3.5 Decoding with Extended Euclidean Algorithm (EEA).....	39	3.6 Decoding with Berlekamp Algorithm (BA).....	43	Chapter IV. Implement Real Program for Algorithm and Results.....	46	4.1 Introductions.....	46
4.2 Flowchart and implementation.....	46	4.3 Results and comparison.....	48	Chapter V. Conclusion and Future Work.....	55	References.....	56	[1] Shu Lin and Daniel J. Costello Jr, Error Control Coding, Pearson Education, New Jersey, 2004. [2]E. R. Berlekamp, Algebraic Coding Theory, McGraw-Hill, New York, 1968. [3]A. Zeh, A. Wachter, and S. Bezzateev, “Efficient decoding of some classes of binary cyclic code beyond the Hartmann-Tzeng bound,” in Information Theory Proceedings (ISIT), 2011 IEEE International Symposium, Aug 2011, pp. 1017-1021. [4] A. Zeh, A. Wachter, and S. Bezzateev, “Decoding Cyclic Codes up to a New Bound on the Minimum Distance,” in Information Theory Proceedings (ISIT), 2012 IEEE International Symposium, Mar 2012, pp. 3951-3960. [5] Jacobus H. Van Lint, Richard M. Wilson, “On the minium Distance of Cyclic Codes,” in IEEE transactions on information theory, Vol. IT-32, No.1, Jan 1986, pp.23-40. [6]Gui-Liang Feng and Kenneth K. Tzeng, “A Generalization of the Berlekamp-Massey Algorithm for Multisequence Shift-Register Synthesis with Applications to Decoding Cyclic Codes,” in IEEE transactions on information theory, Vol. 37, No.5, Sep 1991, pp.1274-1287. [7]Nadia Ben Atti, Gema M. Diaz-Toca, and Henri Lombardi, “The Berlekamp-Massey Algorithm revisited,” Journal Applicable Algebra in Engineering, Communication and Computing, Vol. 17, Arp 2006, pp. 75-82. [8]Jean Louis Dornstetter, “On the Equivalence Between Berlekamp’s and Euclid’s Algorithms,” in IEEE transactions on information theory, Vol. 33, No.3, 1987, pp. 428-431. [9]Ulrich K. Sorger, “A New Reed-Solomon Code Decoding Algorithm Based on Newton’s Interpolation,” in IEEE transactions on information theory, Vol. 39, No.2, Mar 1993, pp. 358-365. [10]C. R. P. Hartmann and K.K. Tzeng, “Decoding Beyond the BCH Bound Using Multiple Sets of Syndrome Sequences,” in IEEE transactions on information theory, Vol. 20, No.2, Mar 1974, pp. 292-295. [11]G. D. Forney, “On Decoding BCH Codes,” in IEEE transactions on information theory, Vol. 11, No.4, Oct 1965, pp. 549-557. [12]J. L. Massey, “Step-by-Step Decoding of the Bose-Chaudhuri-Hocquenghem codes,” in IEEE transactions on information theory, Vol. 11, No.4, Oct 1965, pp. 580-585. [13]C. Roos, “A generalization of the	

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