

# 國小中年級數學課程結構的不確定性

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## 摘要

本研究旨在探討下列議題：(1) 如何以Shannon熵來量化數學課程結構的不確定性？(2) 是否有任何顯著的趨勢發展存在於數學課程結構的不確定性？(3) 學生的數學成就、教師的教學節數誤差值及數學課程結構所對應的Shannon熵之間有何關聯？前兩項議題可透過圖論的方法、Novak概念圖的觀點及所謂的C統計而獲得解決。針對最後一項議題之所需，本研究選取彰化縣公立國小的523位畢業生及18位教師為研究對象。主要研究結果顯示：(1) 數學課程結構的不確定性呈現顯著的遞增現象；(2) 數學課程結構的不確定性和學生的數學成就呈現顯著的負相關；(3) 教師的教學節數誤差值依序與數學課程結構的不確定性及學生的數學成就的相關性皆不顯著。

關鍵詞：不確定性、課程結構、數學成就、Shannon熵

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