

# 摩托車動態與穩定性之研究

武英文、林海平

E-mail: 344755@mail.dyu.edu.tw

## 摘要

在此研究中，使用Lagrange 方程式推導運動方程和其限制條件，在滾動不滑動的條件下，每一個腳踏車輪都有兩個完整和非完整限制條件。使得方程式具有DAE型式。此運動模型已藉由對照基準模型和實驗結果而被驗證其成效。為了解運動方程式中的DAE並確保腳踏車模擬系統中數學的準確性，需要討論其限制條件處理的計算方法。基本上有兩種方法，第一為座標簡化法(coordinate reduction)，第二為內嵌法(embedding method)。另外三種數值法，Baumgarte，後穩定法(post-stabilization)和SMC後穩定法，在此用來修正並提升解題效率。上述方法的成效將被應用，並完成比較其計算時間和數值精確度之結果。當腳踏車改變方向時，騎乘者總是必須去控制腳踏車的側傾角。換言之，控制側傾角為產生轉向或路徑跟隨的首要步驟。在本研究中，使用兩步驟做路徑跟隨控制。第一步驟，在側傾角跟隨控制器中，藉由控制轉向角轉矩，訓練其跟隨參考輸入，確保側傾角在不同情形下的穩定性。第二步驟，發展路徑跟隨控制器以產生適當的參考側傾角，並讓側傾角利用模糊控制器可以跟隨預定義路徑。此控制法則之效果已藉由數學模型所推導的模擬結果證明。此控制法則之成效可用來評估不同之條件，包含：誤差資訊、預路徑設定、固定車速、變動車速和外部干擾因素。

關鍵詞：摩托車動態方程、多體系統、數值穩定、拉格朗日方程

## 目錄

中文摘要.....	iii	ABSTRACT.....	iv
ACKNOWLEDGEMENTS.....	v	TABLE OF CONTENT.....	vi
LIST OF FIGURES.....	viii	LIST OF SYMBOL.....	xi
LIST OF TABLES.....	xii	Chapter I: Introduction.....	1
1.1 Introduction of Two wheel vehicles.....	1	1.1.1 Introduction of Two wheel vehicles.....	1
1.2 Motivation.....	4	1.3 Objective.....	5
1.4 Literature review.....	6	1.4.1 Handling analysis.....	6
1.4.2 Computing method.....	9	Chapter II: Motorcycle model.....	14
2.1 Motorcycle model.....	14	2.2 Basic kinematic theories.....	18
2.3 Kinetic energy.....	20	2.3.1 Kinetic energy of rear frame.....	20
2.3.2 Kinetic energy of front frame.....	23	2.3.3 Kinetic energy of extra terms.....	26
2.4 Potential energy.....	27	2.5 Tire forces and generalized force.....	28
2.5.1 Camber angle and ground steering angle.....	29	2.5.2 Tire forces.....	32
2.5.3 Generalized forces.....	34	2.6 Equations of motion of motorcycle.....	36
2.6.1 Lagrange ' s equation.....	36	2.6.2 Equations of motion.....	37
Chapter III: Stability analysis.....	41	3.1 Eigenvalue method.....	41
3.2 Out-of-plane oscillation.....	42	3.2.1 Capsize mode.....	43
3.2.2 Weave mode.....	44	3.2.3 Wobble mode.....	46
3.3 In-plane oscillation.....	48	3.4 Motorcycle under driving and braking conditions.....	50
Chapter IV: Numerical result.....	53	4.1 Stability of motorcycle under constant speed conditions.....	53
4.2 Stability of motorcycle under driving and braking conditions.....	62	4.2.1 Driving conditions.....	62
4.2.2 Braking conditions.....	65	4.3 Stability of motorcycle affected by the design parameters.....	67
4.3.1 Mechanical trail.....	68	4.3.2 Steering head angle.....	69
4.3.3 Steering damper coefficient.....	71	4.3.4 Height of centre gravity of rear part.....	73
Chapter V: Conclusion and perspectives.....	75	References.....	77
Appendix : Matrix of equations of motion.....	79		

## 參考文獻

[1]Wikipedia, History of the bicycle, available online at: [http://en.wikipedia.org/wiki/History\\_of\\_the\\_bicycle](http://en.wikipedia.org/wiki/History_of_the_bicycle) [2]Wikipedia , Motorcycle history, available online at: [http://en.wikipedia.org/wiki/Motorcycle\\_history](http://en.wikipedia.org/wiki/Motorcycle_history) [3]Whipple, F.J.W., " The stability of the motion of a bicycle, " The

Quarterly Journal of Pure and Applied Mathematics, 1899, Vol.30, pp. 312-348.

[4]Sharp, R.S, " The Stability and Control of Motorcycles, " Journal of Mechanical Engineering Science, 1971, Vol.13(5), pp.316-329.

[5]Sharp, R.S, " Multi-body simulation software in the study of two-wheeled road vehicles, " Conference on Decision and Control, Kobe, Japan, December 1996, pp.2804 - 2805 [6]R.Lot, " A symbolic Approach for Automatic Generation of the Equations of Motion of Multibody Systems, " Multibody System Dynamics, 2004, pp.147-172 [7]Sharp, R.S, " A Motorcycle Model for Stability and Control Analysis, " Multibody System Dynamics, 2001, pp.123 – 142 [8]Limebeer, David J.N, " The stability of motorcycles under acceleration and braking, " Proceedings of the Institution of Mechanical Engineers, 2001, Vol.215(C), pp.1095-1109 [9]Limebeer, David J.N, " Bicycles, Motorcycles and Models, " IEEE Control Systems Magazine, 2006, pp.32-61 [10]Sharp, R.S, " The Influence of the Suspension System on Motorcycle Weave-mode Oscillations, " Vehicle System Dynamics, Vol.5, 1976, pp.147-154 [11]Vittore, Cossater, " Optimum suspension design for motorcycle braking, " Vehicle System Dynamics, Vol.34, 2000, pp.175-198 [12]Lai, Hsien-Chung, " Design parameters study on the stability and perception of riding comfort of the electrical motorcycles under rider leaning, " Elsevier Science Ltd, Vol.13, 2003, pp.49-76 [13]Haim, Baruh, Analytical Dynamics, McGraw-Hill, Singapore, 1999 [14]Hans, B.Pacejka, Tyre and vehicle dynamics, Butterworth Heinemann, London, England, 2002 [15]John, C.Dixon, Tires, Suspension and Handling, 2nd ed., SAE International, Inc. Warrendale, United State of America [16]Vittore, Cossalter, Motorcycle dynamics, 2nd ed., United state of America, 2006 [17]Thomas, D.Gillespie, Fundamentals of vehicle dynamics, 1st ed., SAE International, Inc. Warrendale, 1992 [18]Kelly, S.Graham, Fundamentals of mechanical vibration, 2nd ed., McGraw Hill, Singapore, 2000