

# Degree reduction of NURBS curves by a normalization process

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## ABSTRACT

In order to meet the requirements of various modeling for product appearance, the shapes of the products are no longer lines and arc-shaped. Free-form also becomes a tool for the design shape. In the past, curves used to be formed by many lines while there is very few control points needed to represent this curve in NURBS format nowadays. Therefore, NURBS curves are now applied to replace those tiny lines in the past. It is easy for NURBS curves to adjust the shape we expect. However, the calculation is also relatively complicated. Due to the fact that CNC controller needs more time to deal with higher degree curves, it is usually used 2 and 3. To decrease the amount of time, it is necessary to reduce the higher degree curves to lower degree curves. Therefore, degree reduction has become an important task in CNC machining. This paper proposes a method to deal with the degree reduction for free-form. There are a few steps in this method. First, we apply fitting curves to get a set of initial solution. Secondly, to detect the optimum positions from web-like search algorithm, we have to define global bound error in the process of the algorithm. Thirdly, we have to find the optimum positions through web-like search algorithm. By applying this method, it can effectively reduce the higher degree curves to lower degree curves. This method can apply not only for NURBS curves, but also B-spline and Bezier curves.

Keywords : Degree reduction、Global bound error、NURBS curves、Optimum

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