利用正規化過程處理NURBS曲線降階

蔡易儒、賴元隆

E-mail: 322072@mail.dyu.edu.tw

摘 要

為了滿足顧客對產品外觀的需求，設計產品的形狀不再局限於直線與圓弧，自由曲線也成為設計形狀的工具。過去的曲線是由許多段直線所組成，NURBS曲線只需少許的控制點便可產生此曲線，因此NURBS曲線用來取代過去的曲線。NURBS曲線易於調整形狀直到我們預期的形狀，但是計算過程也相當繁雜。由於 CNC控制器在處理高次曲線需要大量的時間，因此最常使用的次數為2次與3次，為了要減少時間必須將高次曲線降為低次曲線。自由曲線的降階成為CNC加工一個重要的課題。 本論文提出一個方法來處理自由曲線降階的問題。在本方法有幾個主要步驟：第一，利用曲線擬合取得一組初始解。第二，為了使網狀搜尋演算法可以尋找最佳位置，在演算法搜尋的過程藉由定義全域誤差幫助找到最佳解。第三，利用網狀搜尋演算法找到最佳化位置。 透過本方法可以有效地將高次曲線降階為低次曲線。這個方法不僅可以使用在NURBS曲線上，B-spline曲線與Bezier曲線皆可使用。

關鍵詞 : 降階、全域誤差、NURBS曲線、最佳化

目錄

參考文獻

[1]Eck, M. (1995) “ Least squares degree reduction of Bezier curves, ” Comput Aided Des, 27(11):845-861.

[2]Hu, S. M., Wang, G. Z., Jin, T. G. (1996) “ Properties of two types of generalized Ball curves, ” Comput Aided Des, 28(2):125-133.

[3]Brunnett, B., Schreiber, T., Braun, J. (1996) “ The geometry of optimal degree reduction of Bezier curves, ” Comput Aided Geom Des, 13:773-788.

[4]Kim, H. O., Moon, S. Y. (1997) “ Degree reduction of Bezier curves by L1-approximation with endpoint interpolation, ” Comput Math Appl, 33(5):67-77.

[5]Lutterkort, D., Peters, J., Reif, U. (1999) “ Polynomial degree reduction in the L2-norm equals best Euclidean approximation of Bezier coefficients, ” Comput Aided Geom Des, 16:607-612.

[6]Kim, H. J., Ahn, Y. J. (2000) “ Good degree reduction of interval Bezier curves using Jacobi polynomials, ” Comput Math Appl, 40:1205-1215.

[7]Chen, F., Lou, W. (2000) “ Degree reduction of interval Bezier curves, ” Comput Aided Des, 32:571-582.

[8]Chen, G. D., Wang, G. J. (2002) “ Optimal multi-degree reduction of Bezier curves eith constraints of endpoints continuity, ” Comput Aided Geom Des, 19:365-377.

[9]Lee, B. G., Park, Y., Yoo, J. (2002) “ Application of Legendre-Bernsein basis transformations to degree elevation and degree reduction, ” Comput Aided Geom Des, 19:709-718.

[10]Ahn, Y. J. (2003) “ Using Jacobi polynomials for degree reduction of Bezier curves with Ck-constraints, ” Comput Aided Geom Des, 20:423-434.

[11]Ahn, Y. J., Lee, B. G., Park, Y., Yoo, J. (2004) “ Constrained polynomial degree reduction in the L2-norm equals best weighted Euclidean approximation of Bezier coefficients, ” Comput Aided Geom Des, 21:181-191.

[12]Piegl, L., Tiller, W. (1996) “ Algorithm for degree reduction of B-spline curves, ” Comput Aided Des, 27(2):101-110.

[13]Yong, J. H., Hu, S. M., Sun, J. G., Tan, X. Y. (2001) “ Degree reduction of B-spline curves, ” Comput Aided Geom Des, 18:117-127.

[14]Tai, C. L., Hu, S. M., Huang, Q. X. (2003) “ Approximate merging of B-spline curves via knot adjustment and constrained optimization, ”

Comput Aided Des, 35:893-899.
[15]Piegl, L., Tiller, W. (1997) The NURBS book, 2nd ed. Springer, Berlin Heidelberg, New York.
[16]Piegl, L., Tiller, W. (1989)“A menagerie of rational B-spline circles,”IEEE Comput Graph Appl, 9(5):48-56.
[17]David, F., Rogers, J., Alan, A. (1989) Mathematical Elements for Computer Graphics, 2nd edition. McGraw-Hill Higher Education, ISBN:0070535302.
[18]祝華健譯(1990),“電腦繪圖的數學基礎”, 台北:儒林圖書有限公司。